S. Varró, Graphene-based carrier-envelope phase difference meter. Talk presented at the Conference on *Light at Extreme Intensities* (14-18. November 2011. Szeged, Hungary), Book of Abstracts p. 43 (2011).

Graphene–Based Carrier–Envelope Phase Difference Meter

Sándor Varró

Wigner Research Centre for Physics
Hungarian Academy of Sciences, 1525-Budapest, POBox 49, Hungary
E-mail: varro.sandor@wigner.mta.hu

Abstract. In the present paper we report on our theoretical results concerning the reflection and transmission of a few–cycle laser pulse on a very thin conducting layer, which is ment to represent the surface current density of the massless relativistic electrons and holes of graphene. We show that the pulses may undergo violent phase-dependent distortions during the scattering.

Keywords: Nonlinear effects. Graphene.Ultrashort pulses. Carrier–envelope phase difference.
PACS: 41.20 Jb, 42.25Bs, 52.38-r, 73.50.Fq, 78.67.Wj

INTRODUCTION

The generation of phase stabilized few-cycle laser pulses, and the measurement of their absolute phase (carrier–envelope phase difference, or shortly: CE phase) has long been a subject of extensive experimental [1-2] and theoretical [3] research. The real use of extremely short (attosecond) pulses [4], largely relies on their coherence properties, and these can certainly be traced back to the stability of their generating pulses.Thus, it is an important task to find and study processes in which the size of the CE phase plays a significant role and causes measurable effects.

Among various phase-sensitive effects, for example, in reference [3] the high-order above-threshold ionization signal has been proposed as a meter of the absolute phase of the ionizing few-cycle laser pulse. In our recent studies [5-6] we have developed a new theory to describe the reflection and transmission of few-cycle laser pulses on a thin metal nano-layer, and discussed their relevance in measuring the CE phase.

In the present paper we report on new results concerning the reflection and transmission of few–cycle laser pulses on the thin layer of electrons and holes of graphene, which follow ultrarelativistic kinematics. Graphene, the two dimensional one- or few-atom layer of carbon atoms, has many unusual and unique electronic properties, which are well described by massless charged Dirac particles [7-8]. We expect that the electromagnetic response of such a material would be very sensitive to the finest temporal details of ultashort laser pulses, even at moderate intensities.

First, we summarize the basics of our method [5-6], and present the general form of the scattering field configuration; then we derive the relativistic equation of motion for the massless current elements under the action of this composed field, and find a collective radiation damping term, too. Finally, on the basis of numerical solutions, we illustrate the considerable distortions of the radiation during the scattering.

BOUNDARY CONDITIONS OF THE EM FIELDS AT THE GRAPHENE LAYER AS AN 'ACTIVE BOUNDARY'

We analyse, essentially, a special system of the coupled Maxwell-Lorentz equations of the incoming and scattered radiation and the scattering surface current elements [5-6].

The components of a p-polarized configuration of waves $(0, E_y, E_z)$ and $(B_x, 0, 0)$ satisfy the following Maxwell equations in two media, in regions 1 and 3

$$\partial_z B_x = \partial_0 \varepsilon E_y , \quad -\partial_y B_x = \partial_0 \varepsilon E_z , \quad \partial_y E_z - \partial_z E_y = -\partial_0 B_x . \qquad (1)$$

The primary wave propagates (in the $y-z$ plane, making an angle θ_1 with the positive z-axis) towards the interface ($z=0$), which separates the two media of dielectric permittivities $\varepsilon_{1,3} = n_{1,3}^2$. In *region 2, which is the graphene layer itself*, the field configuration generates an induced current, in this region a term $4\pi j_{2y}/c$ has to be added to the rhs of the first equation of Eq. 1. In region 1 we take B_x as a superposition of the *given incoming plane wave pulse* F and an *unknown reflected plane wave* f_1. The corresponding electric field components E_y and E_z can also be derived from Eq. 1.

$$B_{1x} = F - f_1 = F[t - n_1(y\,sin\theta_1 - z\,cos\theta_1)/c] - f_1[t - n_1(y\,sin\theta_1 + z\,cos\theta_1)/c] , \quad (2)$$

$$E_{1y} = (cos\theta_1 / n_1)(F + f_1) , \quad E_{1z} = (sin\theta_1 / n_1)(F - f_1) . \qquad (3)$$

In region 3 the magnetic induction B_{3x} is represented by the *unknown refracted wave* g_3, and, by putting this into Eq. 1, the electric field strength can be obtained,

$$B_{3x} = g_3[t - n_3(y\,sin\theta_3 - z\,cos\theta_3)/c] , \quad E_{3y} = (cos\theta_3 / n_3) g_3 , \; E_{3z} = (sin\theta_3 / n_3) g_3 . \; (4)$$

The relevant boundary conditions for the tangential components read

$$[E_{1y} - E_{3y}]_{z=0} = 0 , \quad [B_{1x} - B_{3x}]_{z=0} = (4\pi / c) K_{2y} . \qquad (5)$$

On the basis of Eqs. 2, 3, 4 and 5, the unknown scattered fields f_1 and g_3 can be expressed in terms of the induced surface current K_{y2} (which is not known, either !),

$$f_1(t') = (1/(c_1 + c_3))[(c_3 - c_1)F(t') - c_3(4\pi/c)K_{2y}(t') , \qquad c_1 = cos\theta_1 / n_1 , \quad (6)$$

$$c_3 g_3(t') = E_{3y}(t') = (2c_1c_3/(c_1 + c_3))[F(t') - (2\pi/c)K_{2y}(t')] , \qquad c_3 = cos\theta_3 / n_3 , \quad (7)$$

where $t' = t - yn_1\,sin\theta_1 / c$ denotes the retarded time parameter at the surface. According to Snell's law of refraction ($n_1\,sin\theta_1 = n_3\,sin\theta_3$) t' must be equal to $t'' = t - yn_3\,sin\theta_3 / c$. Eqs. 6 and 7 are valid for any (constant) $n_{1,3}$, regardless of the nature of K_{y2}. It is interesting to note that in our procedure the surface current in Eq. 5 plays a role of a sort of 'active boundary' contributing to the matched fields. On the other hand, the force term in the equation of motion of this surface current elements must contain the total field (including the unknown scattered one), and in this way, the present formalism automatically accounts for a 'collective radiation damping'.

CLASSICAL EQUATION OF MOTION OF THE MASSLESS CHARGES AND 'COLLECTIVE RADIATION DAMPING'

In our earlier studies [5-6] the *unknown surface current* $K_{2y} = e\eta\delta'_y$ has been expressed as the product of the surface charge density $e\eta$ and the velocity $\delta'_y(t')$ associated to the induced local displacement of the electrons. In the present section we shall proceed similarly; we derive the relativistic equation of motion for the massless current elements under the action of the composed fields, which, in turn are expressed by them, according to Eqs. 6 and 7. Close to the so-called Dirac points in the Brillouin zone of graphene, the transport properties can well be described by a two-dimensional Dirac equation of particles of zero rest mass; a sort of 'charged neutrinos' with an effective speed $\mathrm{v_F} \approx c/300$ [7-9]. By neglecting the radiation reaction we would have

$$\mathrm{v_F}[\alpha_x(-i\partial_x + (e/\hbar c)A_x) + \alpha_y(-i\partial_y + (e/\hbar c)A_y)]\psi = i\partial_{t'}\psi\,, \qquad (8)$$

where the vector potential would represent the incoming (external) field F. By solving Eq. 8, one could calculate the induced surface current $e\mathrm{v_F}\psi^+\boldsymbol{\alpha}\psi$, and express the scattered fields. However, in this procedure the back reaction of the scattered field would not be taken into account, and, moreover, the gauge invariance [9-10] of the results would not be secured. On the other hand, in the classical description to follow we do not encounter with such difficulties.

The equation of motion of a charge element (in the present case p_x=const.) reads

$$\frac{dp_y(t')}{dt'} = \frac{2c_1c_3}{c_1+c_3}\left[eF(t') - \frac{2\pi e^2}{c}\eta \mathrm{v}_y(t')\right], \qquad \mathrm{v}_y = \frac{\partial E(\boldsymbol{p})}{\partial p_y} = \frac{\mathrm{v_F}}{|\boldsymbol{p}|}p_y\,, \qquad (9)$$

where we have used Eq. 7, $K_{2y} = e\eta \mathrm{v}_y$ and the dispersion relation $E(\boldsymbol{p}) = \mathrm{v_F}|\boldsymbol{p}|$ of the massless charge elements. In Eq. 9 the second term in the bracket describes the *collective radiation back-reaction*, which always represents a *friction term*, regardless of the sign of the charge. If one finds the solution to Eq. 9, then (by taking Eqs. 2, 3, 4 and 6, 7) one can determine the scattered fields. We represent the incoming field as

$$F(t') = -\partial^2\Pi_y / c^2\partial t'^2\,, \quad \Pi_y = -(c^2F_0/\omega_0^2)f(t')\,cos(\omega_0 t' + \varphi_0)\,, \quad f(t) = e^{-t^2/2\tau}\,, \quad (10)$$

where Π_y is the Hertz potential of the incoming laser pulse of central frequency ω_0, field strength F_0, CE phase φ_0, and $\tau = \tau_L / 2\sqrt{log\,2}$. In the envelope function $f(t)$, τ_L means the full temporal width at half maximum of the intensity. By introducing the scaled variables $\omega_0 t/2\pi = t/T \to t$ and $\mathrm{v_F}\boldsymbol{p}/\hbar\omega_0 \to \boldsymbol{p}$, Eq. 9 reads as

$$\frac{dp_y(t)}{dt} = \frac{2c_1c_3}{c_1+c_3}\beta_\mathrm{F}\left[\pi R\mu_0 f(t)\,cos(2\pi t + \varphi_0) - \frac{e^2}{\hbar c}\beta_\mathrm{F}\eta\lambda_0^2\frac{p_y(t)}{\sqrt{p_x^2 + p_y^2(t)}}\right], \qquad (11)$$

where $e^2/\hbar c$~1/137 is the fine structure constant, $\beta_\mathrm{F} = \mathrm{v_F}/c$, $R = 2mc^2/\hbar\omega_0$ and $\mu_0 = eF_0/mc\omega_0$ is the usual amplitude of 'dimensionless vector potential'. The electron's mass m is here merely a parameter. In the present problem, *the effective intensity parameter* $\pi\beta_\mathrm{F}R\mu_0$, is by (~ $10^{-2}\times10^{6} = 10^{4}$) *four orders of magnitudes* *larger than the usual* μ_0 for 'ordinary' electrons in optical fields ($\hbar\omega_0 \sim$ 1eV). In the 'friction term' in Eq. 11, the factor $\eta\lambda_0^2$ takes on values $10^4 - 10^5$, because the surface charge density of graphene is around $\eta \sim 10^{12} - 10^{13}/\mathrm{cm}^2$, and $\lambda_0^2 \sim 10^{-8}\mathrm{cm}^2$. Thus, the damping factor is ~ 1/800. We present two illustrative figures displaying the reflected signals at a moderate laser intensity, and for initial electron momenta $p_y(0)=0.001=p_x$.

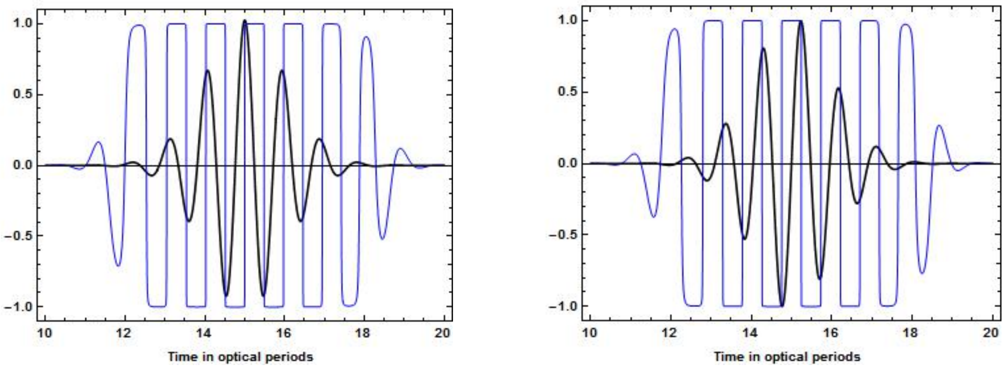

FIGURE 1. Left: shows the normalized electric field strength of an incoming two-cycle *cosine pulse* (thick black line) centered at time t=15T, impinging on the graphene layer at Brewster angle (c_1=c_3 have been chosen in Eq. 11). We have taken $\mu_0=10^{-4}$, corresponding to an intensity of 10 GW/cm^2 at optical frequencies. The thin blue line shows the temporal behavior of the normalized reflected signal whose absolute amplitude is 0.25% of the original one. Right: we have taken an incoming *sine pulse* with the same parameters. The rectangular structure is essentially the same, but here it is shifted by a half cycle.

In conclusion we may say that, according to our description above, the optical response of graphene really causes high nonlinearities. On the other hand, due to this same violent response of the relativistic charges, a very stong radiation damping develops. The interplay of these two effects results in an almost universal rectangular temporal evolution of the reflected signal, which follow the shifts of the CE phase.

ACKNOWLEDGMENTS

This work has been supported by the Hungarian National Scientific Research Foundation OTKA, Grant No. K73728, and by the National Development Agency, Grant No. ELI_ 09-1-2010-0010, Helios Project.

REFERENCES

1. A. Apolonskiy, P. Dombi, G. G. Paulus et al, *Phys. Rev. Letters* **92**, 073902 (2004).
2. S. Witte, R. T. Zinkstok, W. Hogerworst, K. S. E. Eikema, *Appl. Phys. B* **78**, 5-12 (2004).
3. D. B. Milosevic, G. G. Paulus and W. Becker, *Optics Express* **11**, 1418 (2003).
4. F. Krausz and M. Ivanov, *Rev. Mod. Phys.* **81**, 163-234 (2009).
5. S. Varró, *Laser Phys. Letters* **1**, 42-45 (2004) and S. Varró, *Laser Phys. Letters* **4**, 138-144 (2007).
6. S. Varró, *Laser and Particle Beams* **25**, 379-390 (2007).
7. K. S. Novoselov, A. K. Geim, S. V. Morozov et al, *Nature* **438**, 197-200 (2005).
8. A. H. Castro Neto, F. Guinea, N. M. R. Peres et al, *Rev. Mod. Phys*. **81**, 109-162 (2009).
9. S. Varró and J. Javanainen, Gauge-invariant relativistic Wigner functions. *J. Opt. B: Q. S.* 5, S402-S406 (2003)
10. D. Berényi, P. Lévai and V. Skokov, Simulation of pair production in extreme strong fields. *This Proceedings*.